Imposing Differential Constraints on Radial Distortion Correction

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Abstract. Many radial distortion functions have been presented to describe the mappings caused by radial lens distortions in common commercially available cameras. For a given real camera, no matter what function is selected, its innate mapping of radial distortion is smooth, and the signs of its first and second order derivatives are fixed. However, such differential constraints have been never considered explicitly in existing methods of radial distortion correction for a very long time. The differential constraints we claimed in this paper are that for a given real camera, the signs of the first and second order derivatives of the radial distortion function should remain unchanged within the feasible domain of the independent variable, although over the whole domain, or outside of the feasible domain, the signs may change many times. Our method can be somewhat treated as a regularization of the distortion function within the viewing frustum. We relax the differential constraints by using a deliberate strategy, to yield the linear inequality constraints on the unknown coefficients of the radial distortion function. It seems that such additional linear inequalities are not difficult to deal with in recent existing methods of radial distortion correction. The main advantages of our method are not only to ensure the recovered radial distortion function satisfy differential constraints within the viewing frustum, but also to make the recovered radial distortion function working well in case of extrapolation, caused by the features used for distortion correction usually distributed only in the middle part, but rarely near the boundary of the distorted image. The experiments validate our approach.

1 Introduction

The ideal pinhole model is often employed in algorithms of 3D recovery from 2D images in the field of computer vision. Unfortunately, for common commercially available cameras, they usually do not strictly satisfy the ideal pinhole model, i.e., some deviations may exist. Such deviations can be more complex, and are called as lens distortions in literature [1]. There are many methods to model lens distortions. The most famous model was proposed by Brown [1] which described the radial, decentring and prism distortions. In fact, among these distortions, radial distortion is the most significant in recent cameras [2], [3], [4], [5], [6],

[7], [8], [9], [10], [11], [12]. Other types of distortions are often little, and can be omitted in the calibration procedure of distortion correction.

Many kinds of radial distortion functions are presented to describe the radial distortion [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12]. If we assume the center of radial distortion is known in advance, we can define the distance from the original distorted image point to the center of radial distortion as the distorted radius r_d, and the distance corresponding to the undistorted image point as the undistorted radius r_u. The radial distortion functions usually describe the relations between r_d and r_u, namely, $r_d = f(r_u)$ or $r_u = g(r_d)$ [1], [13], [14], [2], [15], [16], [17]. For some cases of metric calibration, the view angle θ corresponding to the undistorted image point is often chosen to replace of r_u. Now, radial distortion functions become $r_d = p(\theta)$ or $\theta = q(r_d)$ [18], [19], [20], [5], [12]. Basu and Licardie [14] proposed the logarithmic distortion model. Devernay and Faugeras [21] presented the field-of-view distortion model. Fitzgibbon [16] recommended the division model with a single parameter. Ying and Hu [3], Barreto and Daniilidis [4] extended the unified imaging model of central catadioptric cameras to describe the radial distortion. Claus and Fitzgibbon [17] constructed the rational function distortion model for a wide range of radial distortions. Hartley and Kang [6] utilized a nonparametric model for radial distortion.

For radial distortion functions of real lenses estimated using different methods [18], [13], [15], [22], [23], [24], [25], [26], [27], [28], [19], [29], [20], we can easily find a phenomenon that, the signs of their first and second order derivatives with respect to the radius (i.e., r_d or r_u) or the view angle (i.e., θ) should remain unchanged within the viewing frustum, namely, from zero to the maximum of the radius or the view angle within the feasible domain. However, such constraints are never considered explicitly in literature. One reason may be that someone may think such constraints can be satisfied automatically. Indeed, without the constraints, the signs may be changed within the feasible domain (Many examples often violated such constraints, such as, Fig. 3 and Fig. 7 in [27], and Fig. 8 in[22]). Another reason may be that to impose such constraints on existing methods of radial distortion correction may bring difficulties to optimizations. However, in fact, we demonstrate that if using a deliberate strategy, such constraints can often be relaxed to the linear inequality constraints on the unknown coefficients of the radial distortion functions. Here, we take the 3-order polynomial $r_u = g(r_d) = k_1 r_d + k_2 r_d^2 + k_3 r_d^3$ as a very easy instance (Other order polynomials can be dealt with in a very similar manner). If the original objective function in some recent existing method for radial distortion correction is $J = J(k_1, k_2, k_3)$, the solution is corresponding to the global minimum of the objective function:

$$\min_{k_1,k_2,k_3} J(k_1, k_2, k_3)$$

If we impose the differential constraints as claimed in this paper, the optimization problem becomes

$$\min_{k_1,k_2,k_3} J(k_1,k_2,k_3)$$
$$\text{subject to } k_1+2r_dk_2+3r_d^2k_3>0 \text{ for all } 0<r_d\leq r_{dmax}$$
$$2k_2+6r_dk_3>0 \text{ for all } 0<r_d\leq r_{dmax}$$

where the first order derivative $g'(r_d)=k_1+2k_2r_d+3k_3r_d^2=k_1+2r_dk_2+3r_d^2k_3$, the second derivative $g''(r_d)=2k_2+6k_3r_d=2k_2+6r_dk_3$, and r_{dmax} is the maximum of r_d in a given distorted image taken by a real camera. The second derivative greater than zero means that the radial distortion is barrel (And the second derivative less than zero is corresponding to pincushion distortion). The above optimization problem seems more difficult to solve. We relax it as follows:

$$\min_{k_1,k_2,k_3} J(k_1,k_2,k_3)$$
$$\text{subject to } k_1+2r_{di}k_2+3r_{di}^2k_3>0, i=1,...,n$$
$$2k_2+6r_{di}k_3>0, i=1,...,n$$

where r_{di} are some sample points lying in between 0 and r_{dmax}. The number of sample points n, can be selected easily as some reasonable number, e.g., 100. In this paper, we simply let $r_{di}=\frac{i}{n}r_{dmax}$, $i=1,...,n$. Obviously, now the constraints become the linear inequalities in the unknown coefficients. For pincushion distortion, it can be solved in a very similar manner.

Recently, the division model with one coefficient proposed in [16] and its extended versions with more coefficients are very popular and often employed for radial distortion correction [17], [5], [7], [8], [9], [11], [30], [31], [32], [10], [33]. However, different from the polynomial models as discussed above, we cannot relax differential constraints in the division models in a direct way, since the constraints derived from differential constraints in the division models are no longer linear inequalities in the unknown coefficients. However, in this paper we use a deliberate strategy to show that such problems still can be relaxed to linear inequalities, which will be discussed in details in the main text.

Furthermore, we take the method proposed in [22] as an instance to show how to relax differential constraints in the extended division models, and what changes may be caused by imposing the additional linear inequalities on the process of optimization. In the original method proposed in [22], the unknown coefficients of radial distortion were solved by finding the Moore-Penrose pseudoinverse of a matrix, i.e., it is a linear method. After imposing the additional linear inequalities, the problem becomes a convex optimization. That means the novel method considering differential constraints can still be solved easily, since there are many software kits for convex optimization. It does not require initial values, and any local minimum must be a global minimum. Especially, by comparing the experimental results from the original method in [22] and our novel method with differential constraints, we can find out that our method is able not only to ensure the recovered radial distortion function satisfy differential constraints within the feasible domain, but also to make the recovered radial distortion function working well in case of extrapolation.

2 The Division Undistortion Model with Differential Constraints

The division undistortion model is firstly proposed by Fitzgibbon [16], which described the relations between r_d and r_u as follows:

$$r_u = g(r_d) = \frac{r_d}{1 + k_1 r_d^2} \tag{1}$$

There is only one coefficient k_1 in the radial distortion function. We can easily find out that, in general, $k_1 > 0$ is corresponding to pincushion distortion, and $k_1 < 0$ is corresponding to barrel distortion. However, from Fig. 1a, we can find that for some pincushion distortion, e.g., $k_1 = 1$, the first order derivative of $g(r_d)$ change the sign where $r_d = 1$. Note that for a real lens, it cannot have such curve.

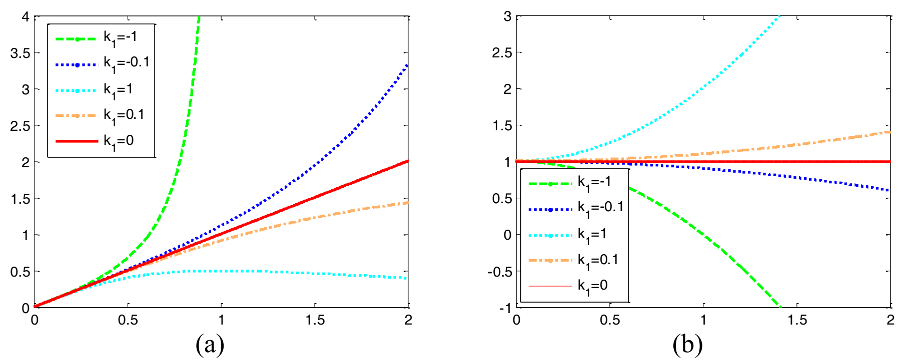


Fig. 1. (a) The curves of r_d vs. r_u under the division undistortion model, i.e., $r_u = \frac{r_d}{1+k_1 r_d^2}$, with different coefficient k_1 (Similar figure is shown as Fig. 2 in [32]). We can notice that for $k_1 = 1$, the sign of the first order differential of the curve changes where $r_d = 1$. Note that for a real lens, it cannot have such curve. (b) The curves for the denominator of the division undistortion model, i.e., $g_1(r_d) = 1 + k_1 r_d^2$ with different coefficient k_1.

The first order derivative of $g(r_d)$ is

$$g'(r_d) = \frac{1 - k_1 r_d^2}{(1 + k_1 r_d^2)^2} \tag{2}$$

and the second derivative of $g(r_d)$ is

$$g''(r_d) = \frac{-2k_1 r_d(3 - k_1 r_d^2)}{(1 + k_1 r_d^2)^3} \tag{3}$$

For barrel distortion, since $k_1 < 0$, $g'(r_d)$ and $g''(r_d)$ are automatically greater than zero in the feasible domain, i.e., $0 < r_d \leq r_{dmax}$ (here we assume the denominator $1+k_1r_d^2$ not equal to zero, indeed such case means that the field of view of the real camera may be larger than 180 degrees, which will be discussed in the next section). Therefore, for barrel distortion with the division undistortion model, the optimization problem becomes

$$\min_{k_1} J(k_1)$$
$$\text{subject to } k_1 < 0$$

For pincushion distortion, $g'(r_d)$ should be greater than zero, and $g''(r_d)$ is less than zero. We cannot relax these differential constraints as the polynomial models described before, since the unknown coefficients of the radial distortion functions are no longer linear inequalities. We notice a fact that, $g'(r_d) > 0$ means that $1-k_1r_d^2 > 0$, since the denominator $(1+k_1r_d^2)^2$ is always greater than zero. Similarly, $g''(r_d) < 0$ means that $k_1 > 0$ and $3-k_1r_d^2 > 0$. Since $1-k_1r_d^2 > 0$ and $k_1 > 0$, must ensure that $3-k_1r_d^2 > 0$, for pincushion distortion with the division undistortion model, the optimization problem becomes

$$\min_{k_1} J(k_1)$$
$$\text{subject to } k_1 > 0$$
$$1-r_{di}^2k_1 > 0, i = 1, ..., n$$

where $r_{di} = \frac{i}{n} r_{dmax}$.

3 The Extended Division Undistortion Model with Differential Constraints

Since the division undistortion model proposed by Fitzgibbon [16] only has one coefficient, it is required to be extended with more coefficients to represent more complex radial distortion functions [5], [7], [8], [9], [11], [30], [31], [32], [10], [33]:

$$r_u = g(r_d) = \frac{r_d}{1+\sum_{i=1}^{m} k_i r_d^{2i}} \tag{4}$$

where $2m$ is the highest degree in the denominator. We indicate the denominator of $g(r_d)$ as

$$g_1(r_d) = 1 + k_1r_d^2 + k_2r_d^4 + k_3r_d^6 \tag{5}$$

Obviously, it is an even-degree polynomial. Some curves of $g_1(r_d)$ corresponding to different m are shown in Fig. 2. Now, we take $m = 3$ as an instance (Other degrees can be dealt with in a very similar manner):

$$r_u = g(r_d) = \frac{r_d}{1+k_1r_d^2+k_2r_d^4+k_3r_d^6} \tag{6}$$

The first order derivative of $g(r_d)$ is

$$g'(r_d) = \frac{1-k_1r_d^2-3k_2r_d^4-5k_3r_d^6}{(1+k_1r_d^2+k_2r_d^4+k_3r_d^6)^2} \tag{7}$$

The second derivative of $g(r_d)$ is

$$g''(r_d) = \frac{-6k_1 r_d + 2(k_1^2 - 10k_2)r_d^3 + (6k_1k_2 - 42k_3)r_d^5 + 12k_2^2 r_d^7 + 34k_2k_3r_d^9 + 30k_3^2 r_d^{11}}{(1 + k_1r_d^2 + k_2r_d^4 + k_3r_d^6)^3} \tag{8}$$

For barrel distortion, since $g'(r_d) > 0$ and $g''(r_d) > 0$ in the feasible domain (here we assume the denominator $1+k_1r_d^2+k_2r_d^4+k_3r_d^6$ not equal to zero, indeed such case means that the field of view of the real camera may be larger than 180 degrees, which will be discussed later), we have

$$\begin{gathered}\min_{k_1,k_2,k_3} J(k_1,k_2,k_3)\\ \text{subject to } 1 - r_{di}^2 k_1 - 3r_{di}^4 k_2 - 5r_{di}^6 k_3 > 0, i = 1,...,n\\ -6r_{di}k_1 + 2r_{di}^3(k_1^2 - 10k_2) + r_{di}^5(6k_1k_2 - 42k_3) + 12r_{di}^7k_2^2 + 34r_{di}^9k_2k_3 + 30r_{di}^{11}k_3^2 > 0\\ i = 1,...,n\end{gathered}$$

where $r_{di} = \frac{i}{n} r_{dmax}$. However, we notice that the constraints from $g'(r_d) > 0$ are linear inequalities in unknown coefficients, but the constraints from $g''(r_d) > 0$ are quadratic inequalities in unknown coefficients. As we know, in general, an optimization problem with quadratic inequalities is not easy to be solved. However, we find that the denominator of $g(r_d)$, i.e., $g_1(r_d)$, its first and second order derivatives, i.e., $g_1'(r_d) = 2k_1r_d + 4k_2r_d^3 + 6k_3r_d^5$ and $g_1''(r_d) = 2k_1 + 12k_2r_d^2 + 30k_3r_d^4$ may satisfy $g_1'(r_d) < 0$ and $g_1''(r_d) < 0$ for barrel distortion (see Fig. 3 in [27]). Therefore, for barrel distortion we have:

$$\begin{gathered}\min_{k_1,k_2,k_3} J(k_1,k_2,k_3)\\ \text{subject to } 1 - r_{di}^2 k_1 - 3r_{di}^4 k_2 - 5r_{di}^6 k_3 > 0, i = 1,...,n\\ 2r_{di}k_1 + 4r_{di}^3k_2 + 6r_{di}^5k_3 < 0, i = 1,...,n\\ 2k_1 + 12r_{di}^2k_2 + 30r_{di}^4k_3 < 0, i = 1,...,n\end{gathered} \tag{9}$$

where $r_{di} = \frac{i}{n} r_{dmax}$. If the field of view of a real camera is larger than 180 degrees, the denominator of $g(r_d)$, i.e., $g_1(r_d)$, can be equal to zero in some r_d corresponding to the view angle equal to 180 degrees, so $g'(r_d)$ and $g''(r_d)$ are undefined on this point. Therefore only $g_1'(r_d) < 0$ and $g_1''(r_d) < 0$ can be used here, i.e., for barrel distortion with the field of view larger than 180 degrees, we have

$$\begin{gathered}\min_{k_1,k_2,k_3} J(k_1,k_2,k_3)\\ \text{subject to } 2r_{di}k_1 + 4r_{di}^3k_2 + 6r_{di}^5k_3 < 0, i = 1,...,n\\ 2k_1 + 12r_{di}^2k_2 + 30r_{di}^4k_3 < 0, i = 1,...,n\end{gathered}$$

where $r_{di} = \frac{i}{n} r_{dmax}$. In Fig. 2b, we show some recovered curves estimated with differential constraints as claimed in this paper.

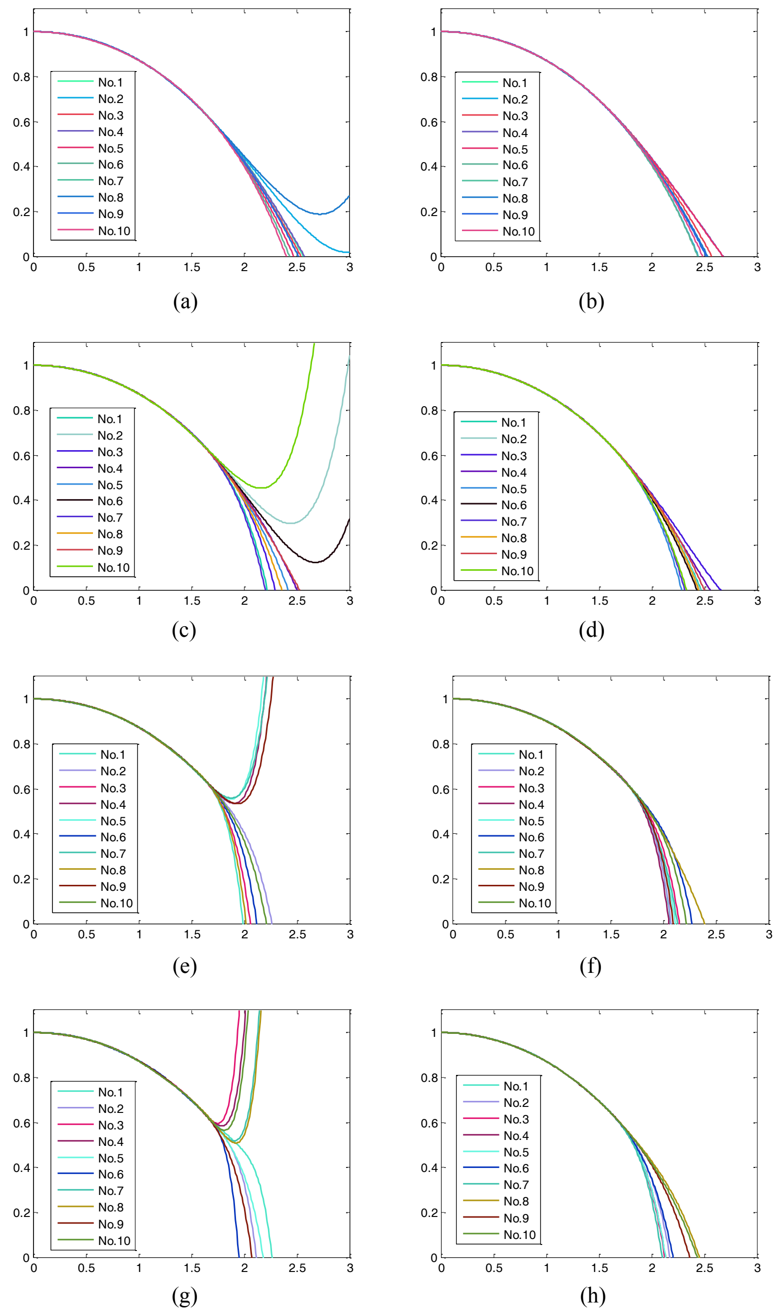


Fig. 2. (a)(c)(e)(g) Some recovered curves without differential constraints, for $g_1(r_d) = \sum_{i=1}^{m} k_i r_d^{2i}$ with different order m, where $m = 3, 4, 5, 6$, respectively (Similar figures are shown as Fig. 3 in [27]). (b)(d)(f)(h) Some recovered curves with differential constraints as claimed in this paper. Note that, the sample points are distributed within from 0 to 1.6. The portions of the curves with greater than 1.6 are the results of extrapolation.

4 Imposing Differential Constraints on Radial Distortion Correction Using the Constraints from the Images of Three Collinear Points

There are so many radial distortion correction methods in literature. Due to lack of space, we only show how to impose differential constraints on the method proposed in [22]. For a distorted image point $(u_d, v_d)^T$, if we establish the origin of the image coordinate system to the center of radial distortion, its corresponding undistorted image point $(u_u, v_u)^T$ under the extended division undistortion model with $m = 3$ may satisfies:

$$\begin{bmatrix} u_u \\ v_u \\ 1 \end{bmatrix} \propto \begin{bmatrix} u_d \\ v_d \\ 1 + k_1 r_d^2 + k_2 r_d^4 + k_3 r_d^6 \end{bmatrix} \tag{10}$$

where " $\propto$ " denotes equality up to a scalar, and $r_d^2 = u_d^2 + v_d^2$. For three undistorted image points $(u_{ui}, v_{ui})^T$, $i = 1, 2, 3$, if they are collinear once rectified, we have

$$\begin{vmatrix} u_{u1} & u_{u2} & u_{u3} \\ v_{u1} & v_{u2} & v_{u3} \\ 1 & 1 & 1 \end{vmatrix} = 0 \tag{11}$$

From (9) and (10), we obtain [22],

$$\begin{vmatrix} u_{d1} & u_{d2} & u_{d3} \\ v_{d1} & v_{d2} & v_{d3} \\ 1 + k_1 r_{d1}^2 + k_2 r_{d1}^4 + k_3 r_{d1}^6 & 1 + k_1 r_{d2}^2 + k_2 r_{d2}^4 + k_3 r_{d2}^6 & 1 + k_1 r_{d3}^2 + k_2 r_{d3}^4 + k_3 r_{d3}^6 \end{vmatrix} = 0 \tag{12}$$

where $r_{di}^2 = u_{di}^2 + v_{di}^2$, $i = 1, 2, 3$. After some manipulations, and let

$$\begin{aligned} a_1 &= u_{d2} v_{d3} r_{d1}^2 - u_{d3} v_{d2} r_{d1}^2 + u_{d3} v_{d1} r_{d2}^2 - u_{d1} v_{d3} r_{d2}^2 + u_{d1} v_{d2} r_{d3}^2 - u_{d2} v_{d1} r_{d3}^2 \\ a_2 &= u_{d2} v_{d3} r_{d1}^4 - u_{d3} v_{d2} r_{d1}^4 + u_{d3} v_{d1} r_{d2}^4 - u_{d1} v_{d3} r_{d2}^4 + u_{d1} v_{d2} r_{d3}^4 - u_{d2} v_{d1} r_{d3}^4 \\ a_3 &= u_{d2} v_{d3} r_{d1}^6 - u_{d3} v_{d2} r_{d1}^6 + u_{d3} v_{d1} r_{d2}^6 - u_{d1} v_{d3} r_{d2}^6 + u_{d1} v_{d2} r_{d3}^6 - u_{d2} v_{d1} r_{d3}^6 \\ b &= u_{d2} v_{d3} - u_{d3} v_{d2} + u_{d3} v_{d1} - u_{d1} v_{d3} + u_{d1} v_{d2} - u_{d2} v_{d1} \end{aligned}$$

we have,

$$a_1 k_1 + a_2 k_2 + a_3 k_3 = -b$$

If there are s triplets of points, we may get s equations as follows:

$$a_{1j} k_1 + a_{2j} k_2 + a_{3j} k_3 = -b_j$$

where $j = 1, ..., s$. Then k_1, k_2, k_3 can be solved as follows:

$$\mathbf{Ax} = \mathbf{b}$$

where

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ & \vdots & \\ a_{1s} & a_{2s} & a_{3s} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} -b_1 \\ \vdots \\ -b_s \end{bmatrix}$$

Therefore, if $s > 3$, we can obtain an overdetermined least squares problem:

$$\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{b}\|$$

The solution is:

$$\mathbf{x} = \mathbf{A}^{+}\mathbf{b}$$

where $\mathbf{A}^{+}$ is the Moore-Penrose pseudoinverse of $\mathbf{A}$ [22]. If impose differential constraints on this optimization problem, we have

$$\begin{gathered}\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{b}\| \\ \text{subject to } 2r_{di}k_1 + 4r_{di}^3k_2 + 6r_{di}^5k_3 < 0, i = 1, ..., n \\ 2k_1 + 12r_{di}^2k_2 + 30r_{di}^4k_3 < 0, i = 1, ..., n\end{gathered}$$

where $r_{di} = \frac{i}{n}r_{dmax}$. The above system represents a sparse convex quadratic program, and can be solved easily using a modern numerical package [34], [35].

5 Experiments

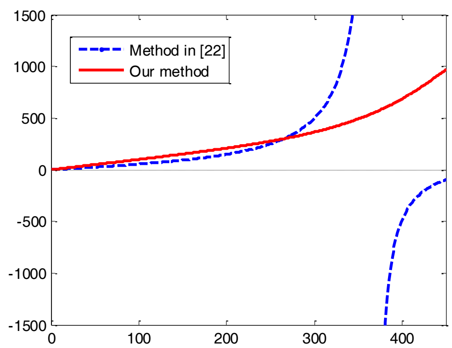
5.1 Simulation

We generate a mapping to simulate real radial distortion, and just select a part of the whole feasible domain of the independent variable to test the performances of methods with (i.e., our method) and without differential constraints (i.e., the method proposed in [22]). Gaussian noise with zero-mean and σ standard deviation is added to sample points of the mapping (Note that, the sample points are distributed within from 0 to 1.6. The portions of the curves with greater than 1.6 as shown in Fig. 2 are the results of extrapolation). We vary the noise level σ from 0 to 1 percent. The estimated results of the radial distortion functions with different degrees are shown in Fig. 2. It is not difficult to find out that our method with differential constraints is very suitable to make the recovered radial distortion function working well in case of extrapolation.

5.2 Real Data

A real image of a corridor was taken by a Basler PIA2400-17gc Color high resolution machine vision camera with Fujinon FE185C046HA-1 Fish-Eye Lens, as shown in Fig. 3a. The resolution of the image is 800×600. From this distorted image, only about 10 sample points on an image curve of a space line are chosen. The corrected image is shown in Fig. 3cd. It is not difficult to find out that the correction results of all images of lines from our method look very reasonable. We also download some images with serious distortion from internet, and one of them is shown in Fig. 4a. We selected about 10 sample points on a single line image curve, and the corrected images are shown in Fig. 4cd. The corrected image from our method also looks very reasonable.

(a)



(b)

(c)

(d)

Fig. 3. (a) A real image of a corridor with radial distortion. (b) The recovered radial distortion functions with (i.e., our method) and without differential constraints (i.e., the method proposed in [22]). (c) The corrected image from the method without differential constraints [22]. (d) The corrected image from our proposed method with the differential constraints of (9). Note that [22] requires ten or more line images in a single image to obtain good results (see Fig. 10 in [22]). However, in this paper, with extremely less requirements, i.e., only some points lying on one line image, we can still obtain satisfactory results. From (b), we may find out that the recovered radial distortion function obtained from [22] maps some image points onto points at infinity, which obviously violated the differential constraints.

6 Discussions

Our approach seems able to be generalized to radial distortion models other than the polynomial and division models as discussed before. We take the distortion model used by Kanatani, Eq (5) in [12] as an example: $\theta = 2\tan^{-1}(c_0 r_d + c_1 r_d^3 + c_2 r_d^5 + \cdots)$. Our problem solving approach is based on a common-sense observation: Generally speaking, $c_0 r_d + c_1 r_d^3 + c_2 r_d^5 + \cdots$ should satisfy the differential constraints, otherwise, $2\tan^{-1}(c_0 r_d + c_1 r_d^3 + c_2 r_d^5 + \cdots)$ may "usually" violate the differential constraints. Note that, "$c_0 r_d + c_1 r_d^3 + c_2 r_d^5 + \cdots$ satisfies the differential constraints", is neither sufficient nor necessary condition for "$2\tan^{-1}(c_0 r_d + c_1 r_d^3 + c_2 r_d^5 + \cdots)$ satisfies the differential constraints". For a polynomial, which satisfies the differential constraints, we can easily convert it into linear inequalities as claimed in Section 1 of this paper. Even though such "simple" "relaxation" is not very "perfect" in mathematics, it seems to seek some better way to implement such differential constraints is very difficult. For other distortion functions, such as, log, sin, and etc., if they contain a polynomial(or even some function with a linear form with respect to distortion coefficients), we can also convert them into linear inequalities in the same manner. Especially, for the extended division model, the polynomial is in the denominator as discussed in Section 3.

7 Conclusions

Radial lens distortion is the most significant one among all kinds of lens distortions in recent cameras. Many models or called radial distortion functions with respect to the radius or the view angle are presented to describe radial lens distortion, and a lots of distortion correction methods are proposed by choosing the suitable one among these different radial distortion functions to accomplish their ideas. However, the differential constraints of these radial distortion functions were omitted in literature for a very long time. In this paper, we suggest the differential constraints should be imposed in the existing methods for radial distortion corrections. The constraints are that the signs of the first and second order derivatives of a radial distortion function with respect to the radius or the view angle should remain unchanged within the viewing frustum, or the feasible domain of the radius or the view angle, namely, from zero to the maximum of the radius or the view angle corresponding to the distorted image, although over the whole domain, the signs may be changed many times. Our method can be somewhat treated as a regularization of the distortion function within the viewing frustum.

To impose differential constraints on radial distortion correction, is very important and useful, since many examples often violated such constraints, such as, Fig. 3 and Fig. 7 in [27], and Fig. 8 in [22]. To impose differential constraints onto existing methods, is not trivial, but usually very, very difficult. We find some feasible approach to relax such differential constraints to linear inequality constraints on the unknown coefficients of the radial distortion functions. We imposed these constraints into the method proposed in [22] as an instance, which changes the original pseudo-inverse based linear method [22] into a novel convex optimization based method. It seems that to impose differential constraints on some existing methods of radial distortion correction may not bring too many difficulties into optimizations. Note that this paper just takes [22] as an example to show that we can implement differential constraints in some existing distortion correction methods. However, in fact, our basic idea is inspired from an observation: Some curves in Fig. 8 of [22], are very flat in some parts, look very strange, and do not satisfy the differential constraints. It is not difficult to incorporate the linear inequalities from differential constraints into the procedure of recovering the distortion center as proposed in [22]. Note that [22] requires ten or more line images in a single image to obtain good results (see Fig. 10 in [22]). However, in this paper, with extremely less requirements, i.e., only some points lying on a single line image, we can still obtain satisfactory results.

8 Acknowledgment

This work was supported in part by NKBPRC 973 Grant No. 2011CB302202, NNSFC Grant No. 61273283, NNSFC Grant No. 61322309, NNSFC Grant No. 91120004, and NHTRDP 863 Grant No. 2009AA01Z329.

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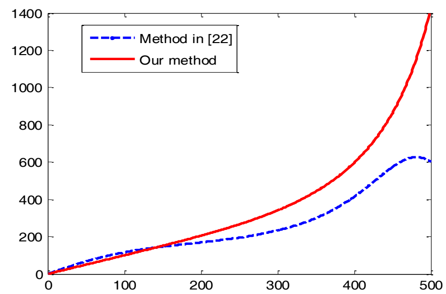
(a)



(b)

(c)

(d)

Fig. 4. (a) A real image of a street scene with radial distortion. (b) The recovered radial distortion functions with (i.e., our method) and without differential constraints (i.e., the method proposed in [22]). (c) The corrected image from the method without differential constraints [22]. (d) The corrected image from our proposed method with the differential constraints of (9). Note that [22] requires ten or more line images in a single image to obtain good results (see Fig. 10 in [22]). However, in this paper, with extremely less requirements, i.e., only some points lying on one line image, we can still obtain satisfactory results.

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